Δ-Y Conversion Proof

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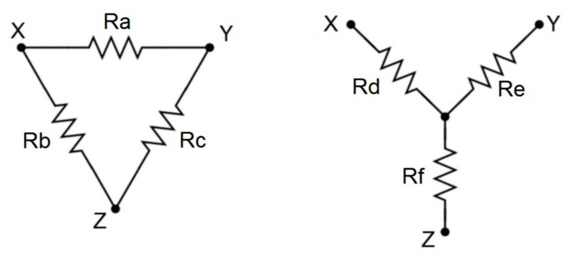
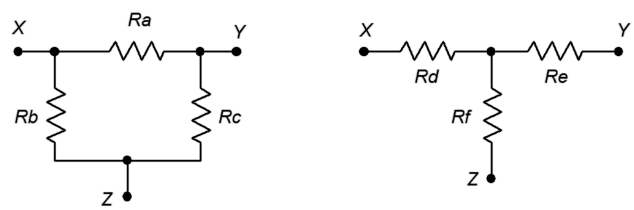
It is possible to convert a Δ (delta) connected three port network into a Y connected three port network and vice versa. Delta networks are also known as pi networks (both in reference to the Greek letters which mimic the shape of the network), and Y networks are also known as T networks. These are shown below.





A true equivalent circuit would present the same resistance between any two terminals as the original circuit. For the unloaded case, we find:

$R_{XY} = R_d + R_e = R_a \parallel (R_b+R_c)$ eq. 1
$R_{XZ} = R_d + R_f = R_b \parallel (R_a+R_c)$ eq. 2
$R_{ZY} = R_e + R_f = R_c \parallel (R_b+R_a)$ eq. 3

Assuming we have the delta and are looking for the Y equivalent, note that we have three equations with three unknowns (R_d, R_e and R_f). Thus, they can be solved using a term elimination process. If we subtract eq. 3 from eq. 1 we eliminate the second resistance (R_e) and arrive at a difference between the first and third unknown resistances (R_d - R_f). This quantity can then be added to eq. 2 to eliminate the third resistance (R_f), leaving just the first unknown resistance (R_d).

$$(R_d + R_e) - (R_e + R_f) = (R_d - R_f) = R_a \parallel (R_b+R_c) - R_b \parallel (R_a+R_c)$$
$$(R_d + R_f) + (R_d - R_f) = 2R_d = 2(R_b \parallel (R_a+R_c) + R_a \parallel (R_b+R_c) - R_c \parallel (R_a+R_b))$$

Therefore,
$R_d = R_b \parallel (R_a+R_c) + R_a \parallel (R_b+R_c) - R_c \parallel (R_a+R_b)$
which, after simplifying, is:
$R_d = R_aR_b/(R_a+R_b+R_c)$ eq. 4

Similarly,
$R_e = R_aR_c/(R_a+R_b+R_c)$ eq. 5
$R_f = R_bR_c/(R_a+R_b+R_c)$ eq. 6

Y-Δ Conversion

For the reverse process of converting Y to delta, start by noting the similarities of the expressions for R_d, R_e and R_f. If two of these expressions are divided, a single equation for R_a, R_b or R_c will result. For example, using eq. 4 and 5:

$$R_d / R_e = (R_aR_b/(R_a+R_b+R_c)) / (R_aR_c/(R_a+R_b+R_c)) = R_aR_b / R_aR_c = R_b / R_c$$

Therefore,

$$R_b / R_c = R_d / R_e$$

$$R_b = R_cR_d / R_e$$

This process can be repeated for eq. 4 and 6 to obtain an expression for R_a. The two expressions for R_a and R_b can then be substituted into eq. 4 to obtain an expression for R_c that utilizes only R_d, R_e and R_f. A similar process is followed for R_a and R_b resulting in:

$$R_a = (R_dR_e + R_eR_f + R_dR_f)/R_f$$

$$R_b = (R_dR_e + R_eR_f + R_dR_f)/R_e$$

$$R_c = (R_dR_e + R_eR_f + R_dR_f)/R_d$$