Series and Parallel Coil Transforms

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Practical Series Coil to Parallel Equivalent

Assume that a practical series coil exists with the components R and jX. What are the values of the equivalent parallel circuit? We shall use A and jB to represent the parallel version and reduce confusion. First, start with the standard parallel formula, and then proceed.

\[
R + jX = \frac{1}{\frac{1}{A} + \frac{1}{jB}}
\]

\[
\frac{1}{R + jX} = \frac{1}{\frac{1}{A} + \frac{1}{jB}}
\]

\[
\frac{1}{R + jX} \cdot \frac{R - jX}{R - jX} = \frac{R}{R^2 + X^2} + \frac{-jX}{R^2 + X^2}
\]

Therefore,

\[
\frac{1}{A} = \frac{R}{R^2 + X^2}
\]

\[
\frac{1}{jB} = \frac{-jX}{R^2 + X^2}
\]

and thus

\[
A = \frac{R^2 + X^2}{R}
\]

\[
jB = j\frac{R^2 + X^2}{X}
\]

If X >> R, then

\[
A \approx \frac{X^2}{R} = Q_{coil}X
\]

\[
jB \approx j\frac{X^2}{X} = jX
\]
Parallel to Series Coil Equivalent

Assume that a parallel inductor network consisting of \( R \) and \( jX \) exists. What is the equivalent series network? Again, we shall name the series components \( A \) and \( jB \) to avoid confusion. First, start with the product-sum rule, and then expand.

\[
A + jB = \frac{RjX}{R + jX}
\]

\[
= \frac{RjX}{R + jX} \cdot \frac{R - jX}{R - jX} = \frac{X^2R}{R^2 + X^2} + \frac{jXR^2}{R^2 + X^2}
\]

Thus,

\[
A = \frac{X^2R}{R^2 + X^2}
\]

\[
jB = j \frac{XR^2}{R^2 + X^2}
\]

If \( R \gg X \), then

\[
A \approx \frac{X^2R}{R^2} = \frac{X^2}{R} = \frac{X}{Q_{\text{parallel}}}
\]

\[
jB \approx j \frac{XR^2}{R^2} = jX
\]