Exponential Charge and Discharge Equations

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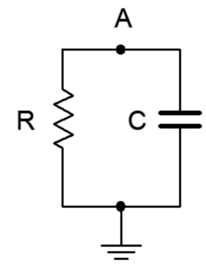
Utilizing a simple series RC circuit, such as depicted in the accompanying figure, we may write an expression for the natural discharge of the capacitor using KCL. We shall assume that the capacitor has some initial starting voltage, $v_C(0) = v_0$. Looking at node A, the total current entering and leaving this node must be 0. Expressing this using the fundamental current/voltage relationships for capacitors and resistors gives us:

$$C\frac{dv_C}{dt} + \frac{v_0}{R} = 0$$



Applying a little algebra:

$$\frac{dv_C}{dt} + \frac{v_0}{RC} = 0$$

$$\frac{dv_C}{v_0} = -\frac{1}{RC}dt$$

Remember, the voltage at $t = 0$ is v_0 and at time t is $v_C(t)$. Integrating will allow us to obtain an expression for $v_C(t)$. Using x and y as variables of integration yields:

$$\int_{v_0}^{v_C(t)} \frac{dx}{x} = -\frac{1}{RC}\int_0^t dy$$

Using the rule regarding natural logs, this can be written as:

$$\ln\left(\frac{v_C(t)}{v_0}\right) = -\frac{1}{RC}t$$

And solving for $v_C(t)$ we discover:

$$v_C(t) = v_0\,\epsilon^{-\frac{t}{RC}}$$

Finally, given that $RC = \tau$ and that the starting voltage for the capacitor normally would be its fully charged value; that of the associated voltage source E, we find:

$$v_C(t) = E\epsilon^{-\frac{t}{\tau}} \qquad \text{(discharge phase)}$$

The equation above also describes the shape of the current (and hence the resistor voltage) during the charge phase, and therefore to satisfy KVL during this phase we find:

$$v_C(t) = E\left(1 - \epsilon^{-\frac{t}{\tau}}\right) \qquad \text{(charge phase)}$$